THE INCREMENTAL REDSHIFT-MAGNITUDE RELATION AND OBSERVATIONAL DATA*

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ABSTRACT

The redshift-magnitude relation expresses a relationship between apparent magnitude and redshift that permits the determination of the density and acceleration parameters from observational data. However, in order to use the redshift-magnitude relation, data must be extended beyond a redshift of one-half and all sources in the set of data used must have about the same intrinsic luminosity. An incremental redshift-magnitude relation is introduced that provides increased sensitivity for the determination of the density and acceleration parameters and the redshift-apparent-magnitude data need only extend beyond a redshift of one-quarter. Sources of different intrinsic luminosity can be used in the same set of data. The incremental redshift-magnitude relation can be interpreted geometrically and provides increased insight into the problem of determining model universes. A method for obtaining the

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incremental apparent magnitude from observational data is presented. Preliminary results of a computing program using the incremental redshift-magnitude relation indicate a trend toward spherical space in which the curvature constant and cosmical constant are positive.

INTRODUCTION

Uniform zero pressure relativistic models of the universe are described by the curvature constant and the cosmical constant. The curvature constant and cosmical constant are functions of the acceleration and density parameters which must be determined from observational data. The usual method to determine these parameters is to apply the redshift-magnitude relation (equation (1) below) to the measured apparent magnitudes and redshifts. The use of the redshift-magnitude relation to determine the acceleration and density parameters requires that all sources in the set of data used in the computation have about the same intrinsic luminosity.

Unpublished results of calculations with the redshift-magnitude relation using data for galaxies from Humanson, Mayall, and Sandage (HMS) 1956 and Baum (McVittie 1965) indicate that observational data should extend beyond redshifts of one-half in order to determine a possible model universe. Unfortunately data for galaxies do not extend beyond a redshift of 0.46. If the redshift of quasi-stellar objects (QSO) is interpreted as arising from the expansion of the universe then the use of the QSO with the larger measured redshifts should provide a better determination of the acceleration and density parameters. However, the use of QSO with galaxies introduces additional problems into the calculation because the intrinsic luminosity of the QSO and galaxies are different.

This paper presents a modification of the redshift-magnitude relation that permits the use of sources with different intrinsic luminosity, increases the sensitivity of the determination of the acceleration and density parameters and lends itself to geometrical interpretation. In addition, a method is given for the determination of the incremental apparent magnitude from observational data. The preliminary results obtained from a program that uses the incremental redshift-magnitude to determine the density parameter and acceleration parameter are discussed.

The Incremental Redshift-Magnitude Relation

The most general form of the redshift-magnitude relation for uniform zero pressure relativistic models of the universe is, Sherman 1965,

$$m = 5 \log_{10}\left\{\frac{c}{H_o} \sqrt{\frac{-k}{1 + q_o - 3\sigma_o}}\,(1 + z)\left(\frac{1}{\sqrt{-k}} \sinh \sqrt{-k}\,\omega\right)\right\} + M - 5 \qquad (1)$$

where

$$\omega = \sqrt{\frac{1 + q_o - 3\sigma_o}{-k}} \int_0^z \frac{dz}{\left\{2\sigma_o z^3 + (3\sigma_o + q_o + 1)z^2 + 2(q_o + 1)z + 1\right\}^{1/2}}$$

In equation (1), m is the apparent magnitude, c is the speed of light, H_o is the Hubble parameter, k is the curvature constant, q_o is the acceleration parameter, σ_o is the density parameter, z is the redshift,

M is the absolute magnitude of the source and the constant 5 comes from the definition of magnitude.

It is of interest to note that the luminosity distance is the argument of the log term and the ratio c/H_o, a constant for a given epoch, is the slope of the luminosity distance with respect to z at $z = 0$. The absolute magnitude of the source, M, is a function of the process taking place in the source and the physical condition of the source; thus, M is independent of z and of the model universe. It is the term in equation (1) that imposes the requirement that all sources have about the same intrinsic luminosity. It is more convenient to write the redshift-magnitude relation as

$$m = 5 \log_{10} \left\{ \sqrt{\frac{-k}{1 \pm q_o - 3\sigma_o}} \, (1 + z) \left(\frac{1}{\sqrt{-k}} \sinh \sqrt{-k}\, \omega \right) \right\} + C \qquad (3)$$

where

$$C = 5 \log_{10} \frac{c}{H_o} + M - 5$$

The three constants (c/H_o, M, and 5) that make up C are independent of the model universe and independent of z. Thus, C is also a constant that is independent of the model universe and z and it contains the term that imposes the requirement that all sources have about the same intrinsic luminosity.

To obtain the incremental redshift-magnitude relation the argument of the log term in equation (2) is multiplied and divided by Z. Equation (2) can now be written as

$$m = 5\ \log_{10} \left\{ \sqrt{\frac{-k}{1 + q_o - 3\sigma_o}} \left(\frac{1 + z}{z} \right) \left(\frac{1}{\sqrt{-k}} \sinh \sqrt{-k}\ \omega \right) \right\} + 5\ \log_{10} z + C \qquad (3)$$

where the last two terms are the linear form of the redshift-magnitude relation. In the linear redshift-magnitude relation, $\sigma_o = q_o = 1$. The linear apparent magnitude, m_l, is given by

$$m_l = 5\ \log_{10} z + C \qquad (4)$$

Subtracting equation (4) from equation (3) gives the following expression for the incremental redshift-magnitude relation

$$\Delta m = 5\ \log_{10} \left\{ \sqrt{\frac{-k}{1 + q_o - 3\sigma_o}} \left(\frac{1 + z}{z} \right) \left(\frac{1}{-k} \sinh \sqrt{-k}\ \omega \right) \right\} \qquad (5)$$

where

$$\Delta m = m - m_l \qquad (6)$$

As the constant C has been eliminated from equation (5), the requirement that all sources have about the same intrinsic luminosity is relaxed. The incremental redshift-magnitude relation refers all uniform zero-pressure relativistic models of the universe to a standard reference, the model represented by the linear redshift-magnitude relation in which $\sigma_o = q_o = 1.0$. When $\sigma_o = q_o = 1.0$ is substituted into equation (5), $\Delta m = 0$ for all z. Equation (5) is valid for zero pressure relativistic models of the universe; however, model universes based on other theories of gravitation or approximate forms of equation (5) can be compared directly with relativistic models if they are reduced to the same standard. See Sherman 1966 for an example in which Robertson's approximate redshift-magnitude relation is compared with equation (5) to determine regions of usefulness.

Geometric Interpretation of the Incremental Redshift-Magnitude Relation

The solid of model universes.- A study of the incremental redshift-magnitude relation indicated that it could be interpreted as a geometric solid, see figure 1, in the Δm, q_o, z coordinate system. This solid called the solid of model universes can extend to infinity in the z and q_o directions and only a small portion of it, the region of interest in the neighborhood of $q_o = 1.0$, is shown in figure 1. The solid can be thought of as a stack of surfaces on which the density parameter and, consequently, the density is constant. The traces of these surfaces on the q_o = constant sections of the solid carry all the information necessary to describe possible models of the universe. Because each curve represents a possible model of the universe the traces of the σ_o = constant surfaces in the q_o = constant sections are called model curves.

The top surface of the solid of model universes is the surface on which density is equal to zero. Because all physically significant models of the universe, models in which $\sigma_o > 0$, lie below the zero density surface the Δm associated with this surface is the upper boundary for Δm. There is no well defined lower boundary for Δm. The lower boundary is a function of the density parameter and is set by the maximum allowable value of σ_o or a value of σ_o that is selected so that undue restrictions are not placed on possible models of the universe. In the work reported herein a σ_o of 6.0 was adopted as not being too restrictive. The Δm associated with the surface on which $\sigma_o = 6.0$ is the lower boundary for the solid of model universes shown in figure 1.

Information content of the solid of model universes.- All physically significant models of the universe of the class considered, those models with $\sigma_o > 0$, lie below the zero density surface. These models of the universe can be described by the curvature constant k and the cosmical constant Λ. As k can be +1, -1, or 0 and Λ can be equal to, greater than, or less than zero there are nine possible combinations of the constants; each combination represents a family of models of the universe. Specific models within these families are identified by the numerical value of Λ. By finding the location of the surface on which $k = 0$ and $\Lambda = 0$ the location of the various families of model universes within the solid of model universes can be determined. The surfaces on which $k = 0$ and $\Lambda = 0$ divide the solid of model universes into four mutually exclusive regions each occupied by one of the combinations of k and Λ, see figure 1. Of the remaining five combinations, four occur on the $k = 0$ and $\Lambda = 0$ surfaces. The remaining combination of k and Λ, $k = \Lambda = 0$ is found at the intersection of the $k = 0$ and $\Lambda = 0$ surfaces. Fourteen families of model universes of the type considered in this paper, see Robertson 1933, are distributed in these regions of the solid or on the surfaces. Each region or surface is singly occupied except the $k = +1$, $\Lambda > 0$ region; and this region is multiply occupied. In the region where $k = +1$ and $\Lambda > 0$ there are an oscillatory model, three types of expanding model, the Einstein stationary model, and a model that is asymptotic to the Einstein model. The families of model universes are identified by the combination of k and Λ associated with the region they occupy. The $k = +1$ and $\Lambda > 0$ designation for the multiply occupied region will not be broken down into specific families.

Sections of the solid of model universes with z held constant are of interest as they indicate the density ranges that can be expected for the various families of model universes. A section of the solid for $z = 1.0$ is shown in figure 2. In addition to the $k = 0$ and $\Lambda = 0$ curves, the traces of the σ_o = constant surfaces are shown. As σ_o is directly proportional to density, McVittie 1965, the density ranges for the various families of models can be estimated. This figure also permits the type of model universe for pairs of σ_o and q_o to be determined.

Extraction of Δm From Observational Data

In order to make use of the incremental form of the redshift-magnitude relation it is necessary to extract a Δm from observational data. It was assumed that the observed apparent magnitudes were composed of three parts, a linear part, a nonlinear part, and an uncertainty due to noise. The method used separates the apparent magnitudes into two parts, the linear part and the nonlinear part plus the noise. The latter part is called Δm_{obs}. Because the observational data for galaxies and QSO divided into several groups of different intrinsic luminosity, the following semigraphical method was used to obtain Δm_{obs}. The method of least squares was used with equation (4) and the observational data for galaxies to obtain the best linear fit to the data. It was found that the variance of the observed m from the fit was 0.15264 and the standard deviation was 0.363. The constant C was found to be 21.402 ± 0.054. Equation (4) was used with this information to construct a grid on which observational data could be plotted. The first step was to plot the line corresponding to $m_l = 5 \log_{10} Z + 21.402$, the solid line labeled 21.402, see figure 3. With this line as a starting point alternate dashed and solid lines were drawn displaced from each other by the standard deviation of m from the fit until a grid covering the spread of observational data had been constructed. Observational data from HMS 1956, McVittie 1965, and Sandage 1965, was superimposed on the grid as shown in figure 3. The observed objects falling between two dashed lines were assigned the value of the constant associated with the solid line that passed between the dashed lines.

After the constant had been determined for each group objects from figure 3, equation (4) were used to compute m_l. Equation (6) was then used to obtain Δm for each source. Figure 4 is a plot of the incremental apparent magnitude obtained from observational data plotted against Z.

There are uncertainties connected with Δm_{obs}. These uncertainties occur because of errors in apparent magnitude, redshift and the constant. In estimating the uncertainty in Δm_{obs} errors in the redshift were assumed to be negligible. The error in apparent magnitude was taken as ± 0.025, de Vaucouleurs 1961, and the uncertainty was ± 0.054. These component uncertainties gave an estimated uncertainty in Δm_{obs} of ± 0.079.

Discussion of the Solid of Model Universes and Observational Data

The range of z.- Model curves for several values of q_o are shown in figure 5. As the redshift approaches zero the model curves approach a focus along the line $\Delta m = z = 0$. The effect of this focusing is to draw the model curves together and between $z = 0.25$ and 0 it becomes almost impossible to resolve the model curves.

With the scatter that is present in available observational data, see figure 4, a great many model curves will fit the data equally well and the selection of a model universe is impossible. This lack of definition between the model curves requires that redshift data extend beyond 0.25 before meaningful results with regard to a model of the universe can be obtained. From this information it can be concluded that the minimum required range of z for the incremental redshift-magnitude relation is 50 percent of that for the redshift-magnitude relation.

Use of simplified models.- Simplified models, like the zero density model, are sometimes used to determine the acceleration parameter. The use of these simplified models can be misleading. Suppose the zero density model was used with a set of observational data to determine q_o which was found to be 4.0. If a σ_o is assumed and used with this q_o the model curve is depressed below the best fit surface, see figure 5, and the new model curve no longer fits the data. With the σ_o included in the model used to determine q_o the best fit curve would lie to the left of $q_o = 4.0$ in figure 2. Experience with exact and zero density fits to the same data have shown wide variation in the q_o's obtained. In one case the zero density model gave a q_o of about 9.0 and the exact model the q_o was 0.262 and σ_o was 2.63. These results indicate that the use of simplified models is dubious and that σ_o and q_o should be determined as a pair from observational data.

Prediction of general class of model.- The solid of model universes makes it unnecessary to use a computer to obtain an indication of the type of model universe indicated by the observational data. As shown in figures 1, 2, and 5, the surfaces of constant σ_o do not maintain the same shape as q_o varies from 5.0 to -1.0 but are warped. Knowledge of warpage of these surfaces when combined with the distribution of Δm_{obs} with z give an important indication of the type of model universe to be expected. The Δm_{obs} shown in figure 5 is clustered near the $\Delta m_{obs} = 0$ axis. Of the 54 points, 21 Δm_{obs} are positive and lie in the range $0 < \Delta m_{obs} \leq 0.33$ and there are 33 points with negative Δm_{obs} that lie in the range $0 > \Delta m_{obs} \geq -0.367$. Thus, the observational results lie between two planes that intersect the Δm axis at 0.33 and -0.367. In order to fit the observational data the model curve must lie between these two planes out to a redshift of 2.0. The distribution of Δm_{obs} with z indicates a curve that has positive slope that changes to a small negative slope near $z = 0.15$ and maintains this slope to the end of the observational data. Through the use of figure 5 it was found that curves with these characteristics occur for $q_o \leq 1.0$. However, the observational data indicate the curve should lie close to q_o, z plane and for $q_o < 0$ the curves show too sharp a positive maximum and the slope after this maximum is too great for the data in figure 4. Thus, the model will be found between $q_o = 0$ and $q_o = 1.0$ and should lie close q_o, z plane. At $z = 1.0$ the model curve should be slightly below this plane. Figure 6 is the solid of model universes with the two boundary planes for the observational added. The hatched area between the negative boundary of the observational data and the q_o axis is the region where the model that fits the observational data shown in figure 4 will be found. The σ_o data presented in figure 2 show that this is a region of medium to high density.

Because the solid of model universes permits the model indicated by the data to be isolated much better initial values of σ_o and q_o, for the iteration process to determine these parameters, can now be selected.

Use of the Incremental Redshift-Magnitude Relation to Analyze Δm_{obs}

A computer program to study the use of the incremental redshift-magnitude relation has just been undertaken. This program makes use of the method of differential corrections, Nielsen 1964, to determine σ_o and q_o so that the root mean square of the Δm residuals is minimized. So far it has been found that use of the solid of model universes with Δm_{obs}, as discussed in the last section, improves the selection of initial values for the iteration process so that the path to the minimum is shorter. The incremental redshift-magnitude relation has been found to be about twice as sensitive as the standard redshift-magnitude relation when used to determine σ_o and q_o from observational data.

Problem areas.- To date two problem areas have been found and are currently being investigated. The first problem area involves solutions where the density parameter is negative. Negative values of σ_o are mathematically acceptable to equation (5) and occur for certain distributions of Δm_{obs}. Solutions with negative density are physically unacceptable. The second problem area involves the weighting of the observational data. Results to date indicate the σ_o and q_o now being computed are heavily affected by the data at large redshifts.

Indicated models of the universe.- A byproduct of this computing program is the determination of σ_o and q_o and, consequently, a model of the universe. When the data shown in figure 4 for galaxies and QSO was analyzed by the use of equation (5), σ_o was found to be about 1.1 and q_o about 0.94. The combination of σ_o and q_o falls in the region predicted through the use of the solid of model universes. This combination of σ_o and q_o indicates a model universe that has $k = +1$ and $\Lambda > 0$, with a density about 2.0×10^{-29} g/cc. This combination of k and Λ means that space is spherical. The use of Robertson's methods, Robertson 1933, shows that a $\sigma_o = 1.1$ and $q_o = 0.94$ described a Le Maitre type universe. As yet there is some scatter in observational data and selection is present in the data; in addition, the sample is still very small with respect to the totality of the problem. Addition or deletion of sources still produce changes in the results. Lastly, there is doubt that the redshift of the QSO is due to the expansion of the universe. In this paper the redshift of the QSO has been assigned to the expansion of the universe so that QSO could be used to illustrate the use of sources of differing intrinsic luminosities. For these reasons, the σ_o and q_o determined from present data should be taken as representing a possible trend rather than a specific model of the universe.

As yet no analysis has been made of the separate groups of sources indicated in figure 3 because of the small number of sources in most of the families. However, the data for galaxies and QSO have been analyzed separately. The results for galaxies indicate a very low density hyperbolic model and the results for the QSO are in very good agreement with σ_o and q_o given above. Too much significance should not be attached to this result because the data for galaxies are a mixture of photographic and photoelectric data and the quality of these data for galaxies is not considered comparable to the quality of the QSO data. In addition, the manner of handling the data for galaxies may have influenced this result. In the Δm_{obs} given in figure 4 the value of the constant for galaxies was read from figure 4 and used even though it was different from 21.402. When the C for all galaxies was taken as 21.402, the results for galaxies and QSO separately and together gave the same type of model universe. For this combination of the data for the galaxies and QSO, $\sigma_o = 1.2$ and $q_o = 1.13$.

In this case, the curvature constant k and cosmical constant Λ are both positive. This result differs little from the previous model in which $\sigma_o = 1.1$ and $q_o = 0.94$ which indicates that the QSO are dominant in determining the model universe.

Before definite conclusions can be drawn more high-quality data, in particular for galaxies, are needed. It would be helpful in the weighting problem if the probable error for each magnitude is known.

CONCLUDING REMARKS

An incremental form of the redshift-magnitude relation has been derived. The incremental redshift-magnitude relation permits use of sources of different intrinsic luminosities in the same group of data, gives increased sensitivity to the determination of σ_o and q_o, and lends itself to geometrical interpretation that gives increased insight into the problem of determining a model universe from observational data. A method for determining an incremental apparent magnitude from observational data has been presented.

Because of the uncertainties connected with observational data, results obtained from a computer program indicate a trend rather than a firm model of the universe. The results obtained to date indicate that curvature constant and cosmical constant are positive and that the density is on the order 10^{-29} g/cc.

More and better quality data with a known probable error are needed before more definitive results can be obtained.

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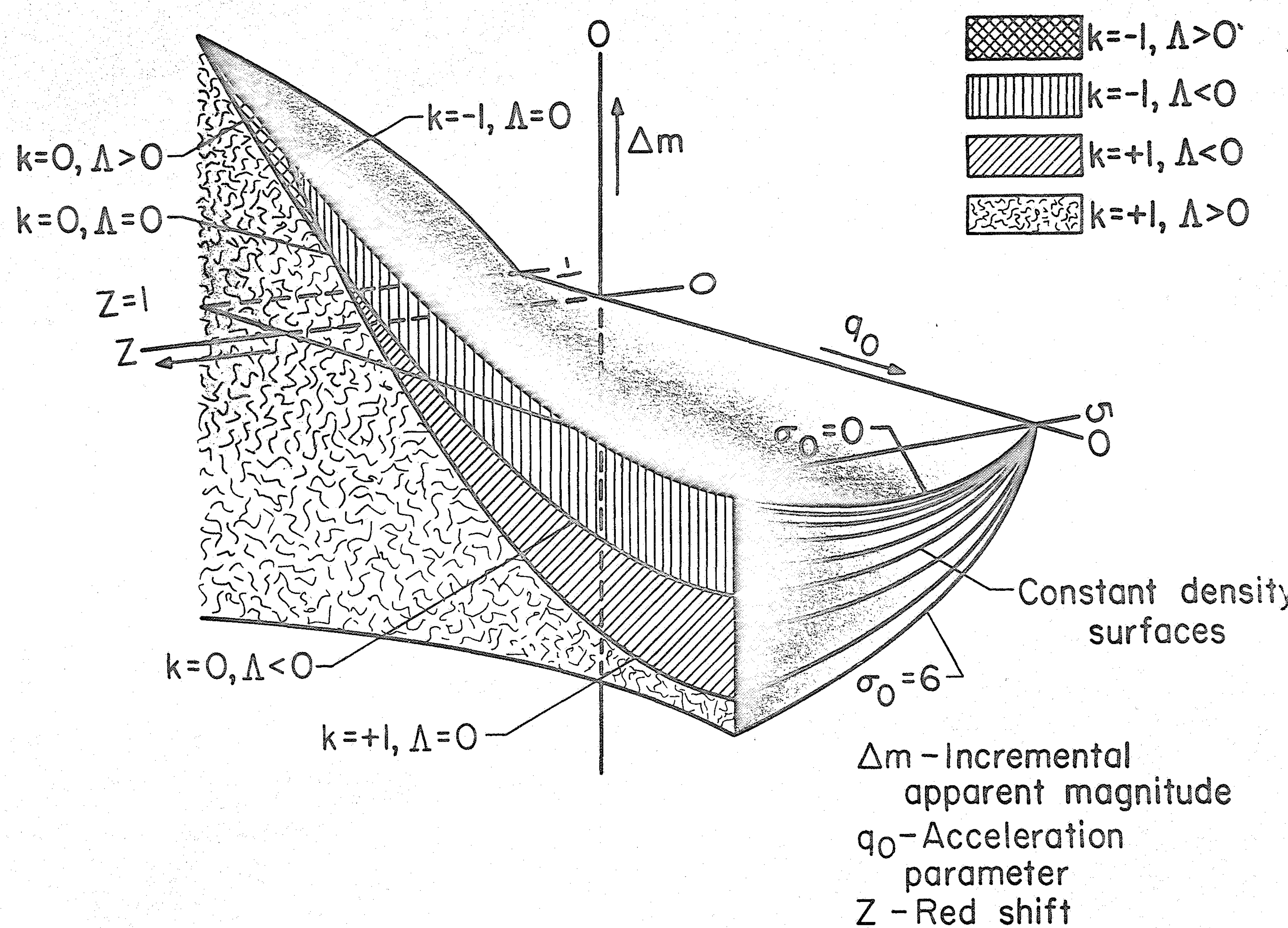

Figure 1.- Portion of the solid of model universes between $q_o = -1.0$ and $q_o = 5.0$ and from $Z = 0$ to $Z = 1.0$. The location of families of models of the universe are shown on the $Z = 1.0$ face, k and Λ have been used to designate the location of the families of models.

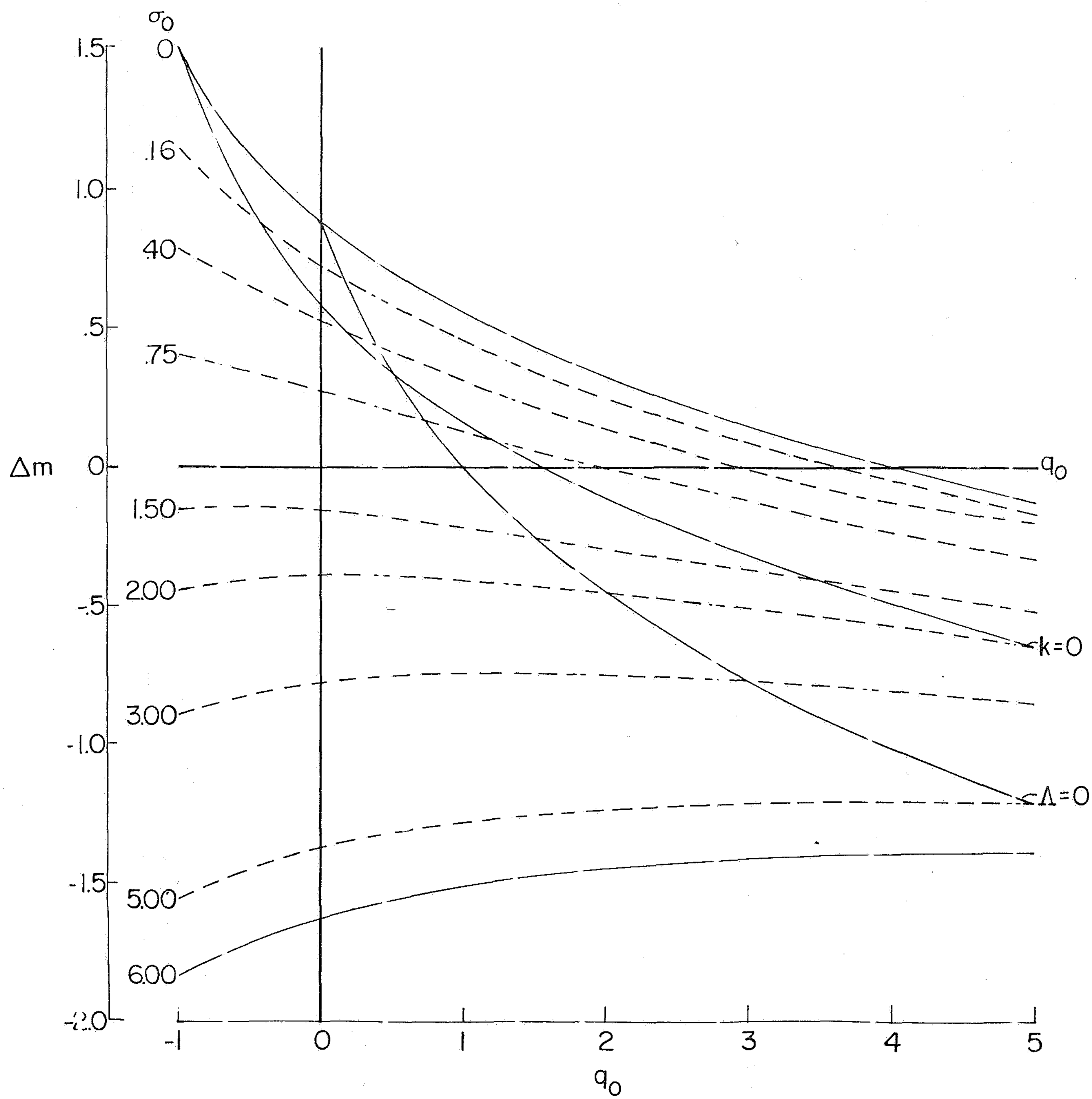

Figure 2.- The $z = 1$ plane of the solid of model universes with surfaces of constant σ_0 superimposed on the division of the solid by the $\Lambda = 0$ and $k = 0$ surfaces.

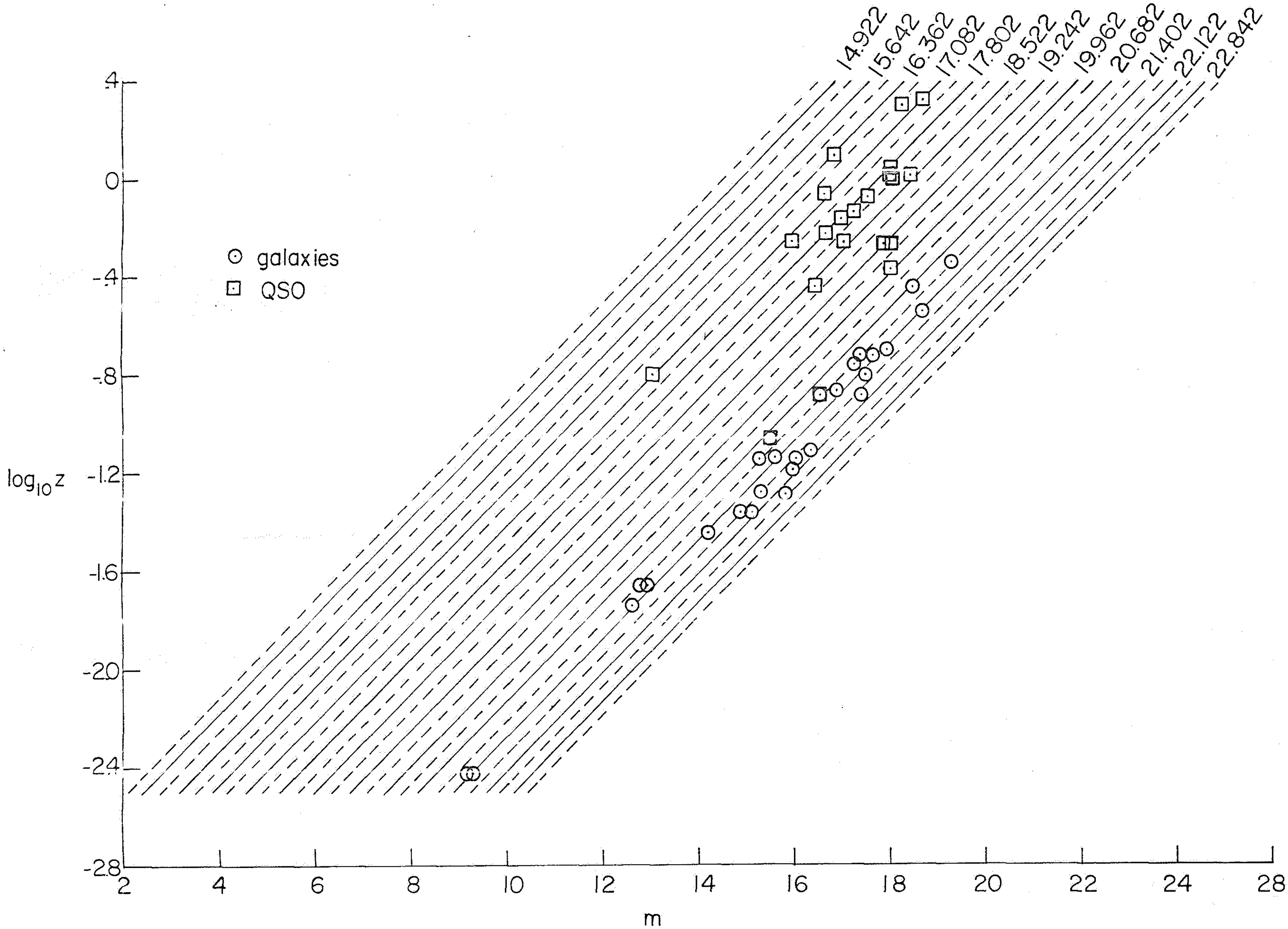

Figure 3.- Finding chart for the constant C. Observational data from (HMS 1956, McVittie 1965, and Sandage 1965) is plotted on this chart.

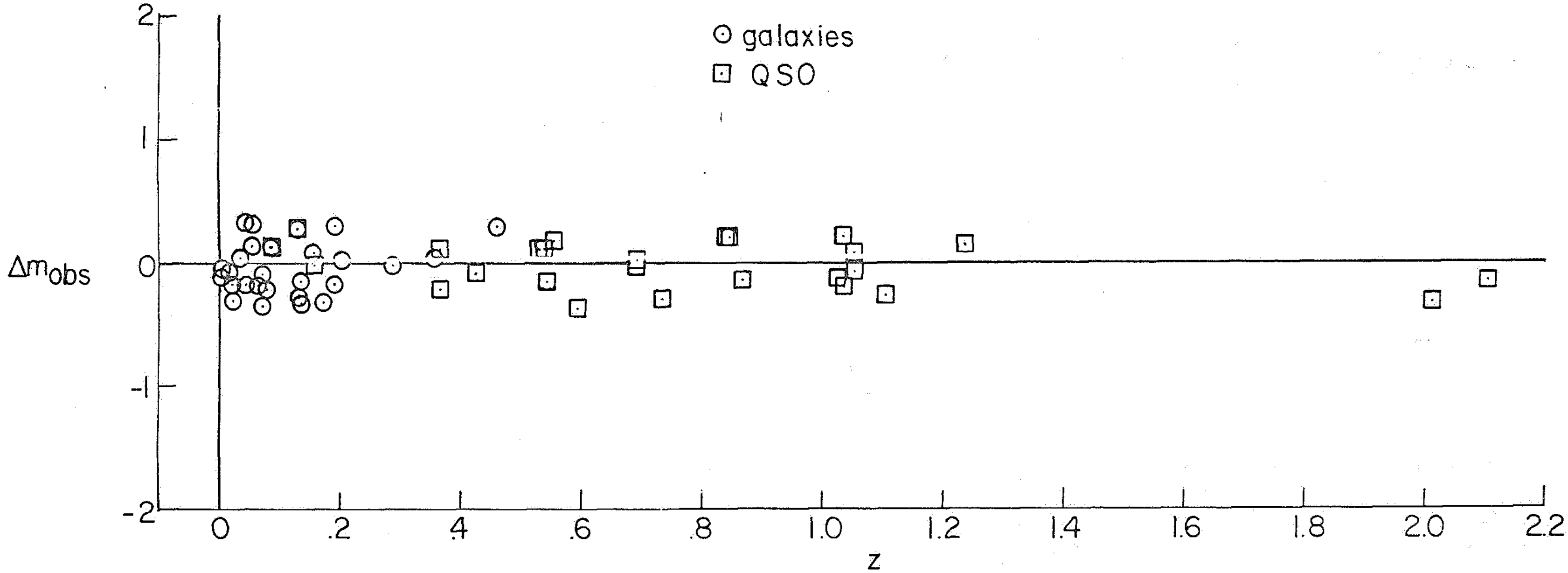

Figure 4.- The distribution of the incremental apparent magnitude, Δm_{obs}, obtained from observational data with redshift, z. Δm_{obs} was plotted against z rather than $\log_{10} z$ so the plot would be compatible with the solid of model universes.

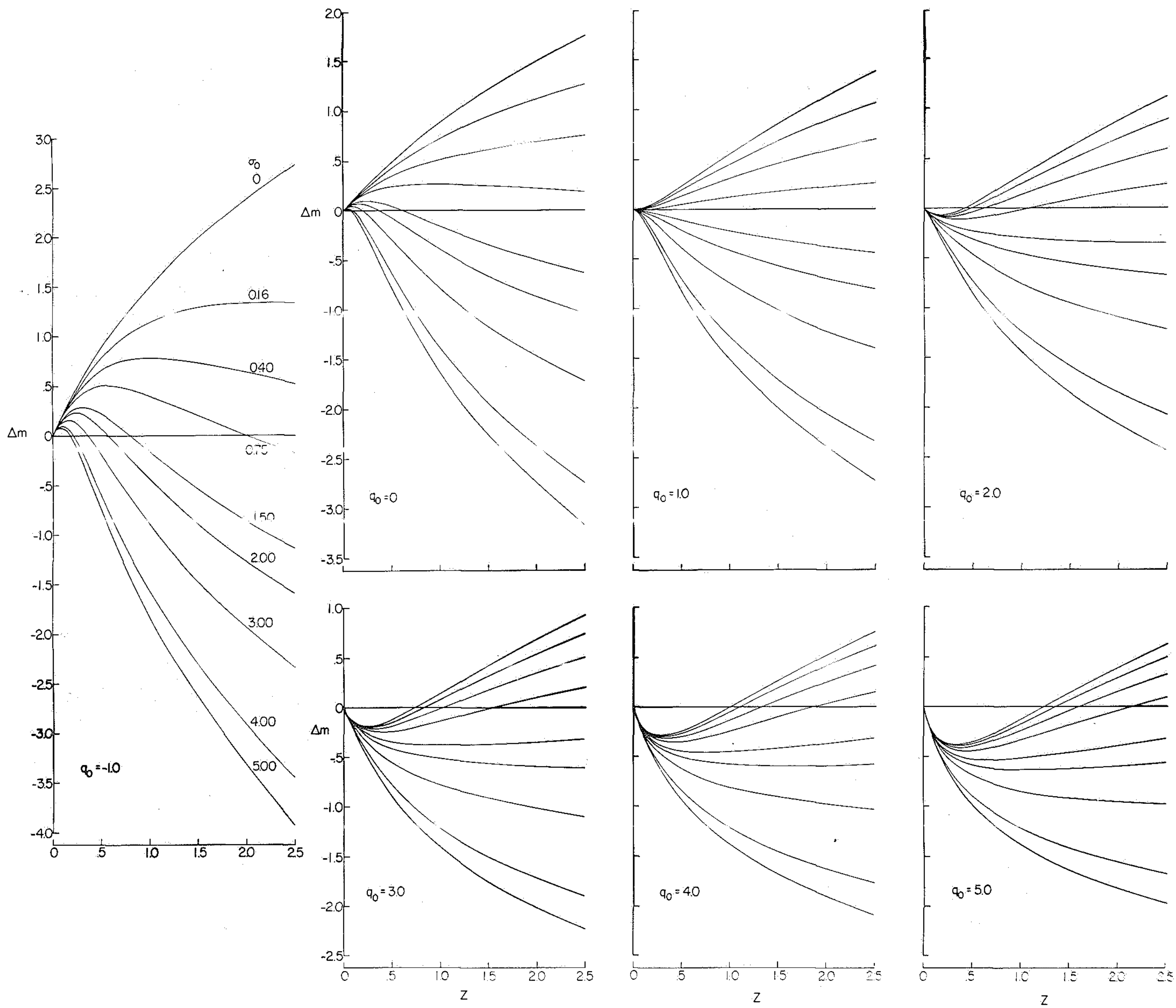

Figure 5.- Sections of the solid of model universes for several q_0 locations. Model curves are shown for several values of σ_0 from $Z = 0$ to $Z = 2.5$ for each q_0.

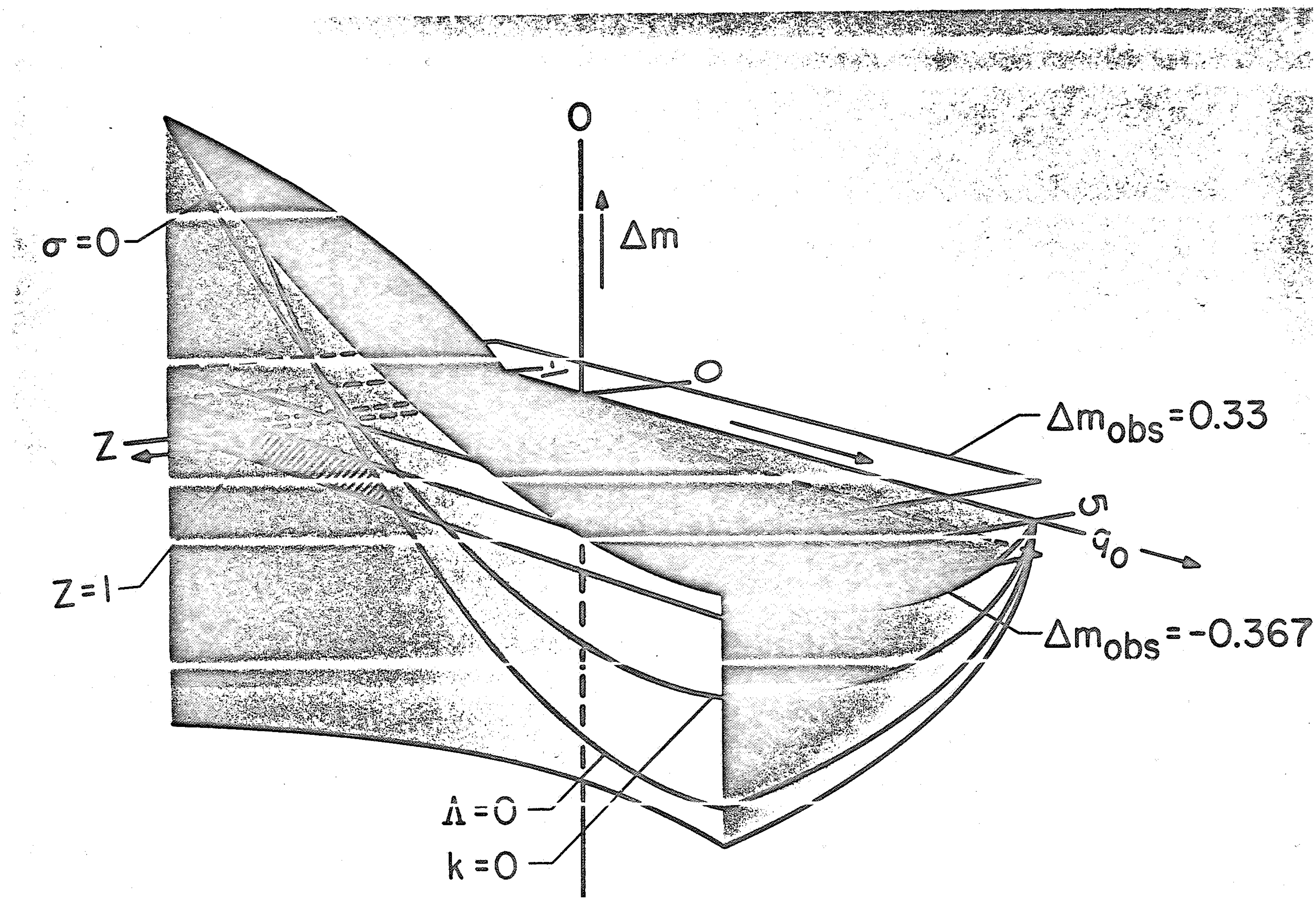

Figure 6.- The solid of model universes and observational data. The two planes that bound the observational data and intersect the Δm axis at $\Delta m = -0.367$ and $\Delta m = 0.33$ are shown. The expected location of the model defined by the observational data is shown by the hatched area on the $Z = 1.0$ face of the solid. The portion of the solid of model universes shown here is the same as that shown in figure 1.